

## Balanced Binaries – AVL Trees

Height of a tree node:

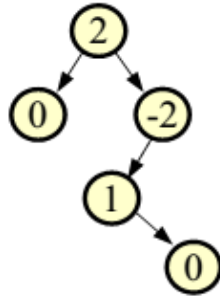
1. The height of a node with no elements is 0
2. The height of a node with 1 element is 1
3. The height of a node with  $> 1$  element is  $1 +$  the height of its tallest subtree

AVL tree:

A binary tree in which the difference between the height of the right and left subtrees of the root is never more than one.

Each node keeps a balance number which is the difference in heights of its two subtrees.

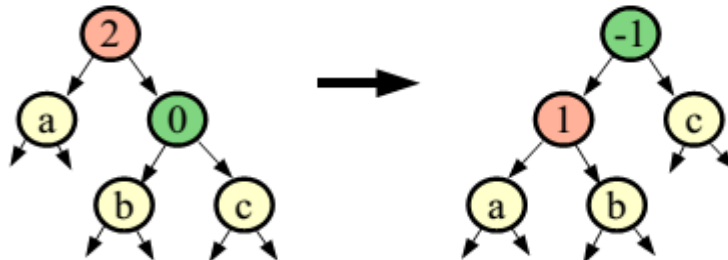
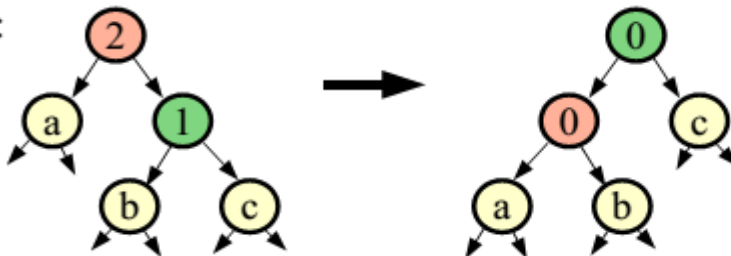
For example,



Whenever a balance number is not 0, -1, +1, perform some rotations according to some rules on following pages

Rules for rotation:

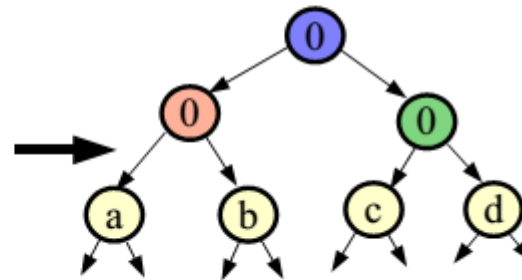
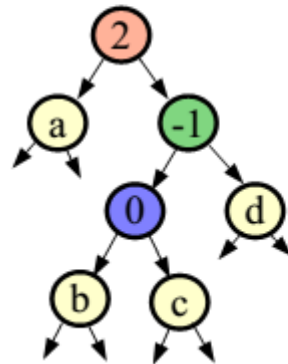
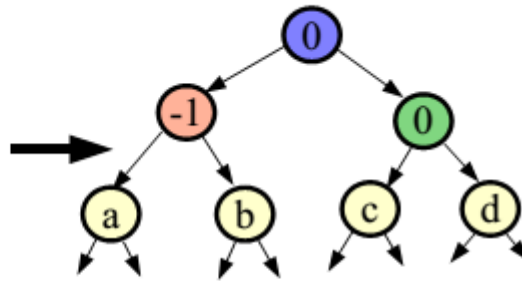
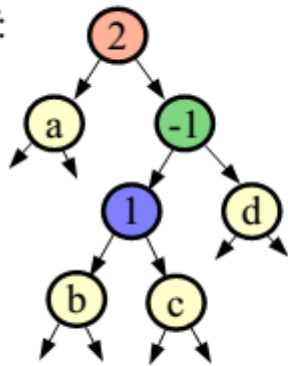
If:



Plus mirror image of these two cases

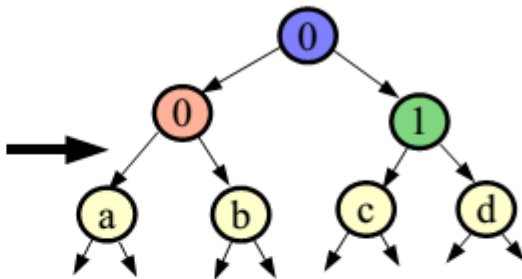
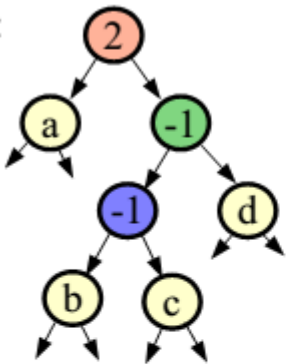
Rules for rotation:

If:



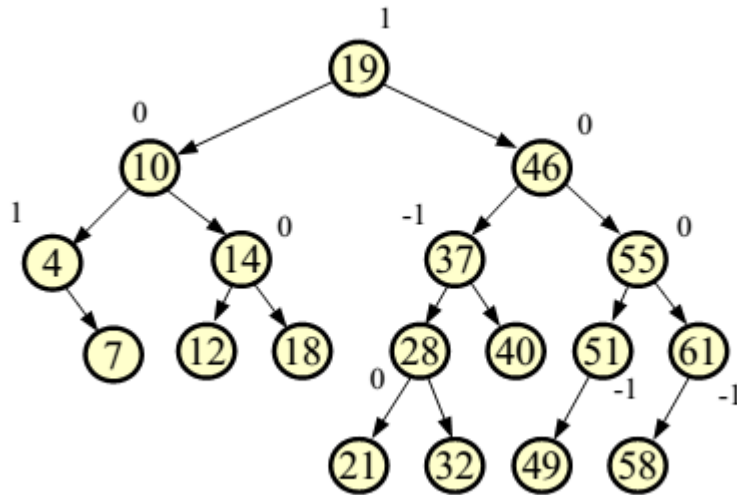
Rules for rotation:

If:

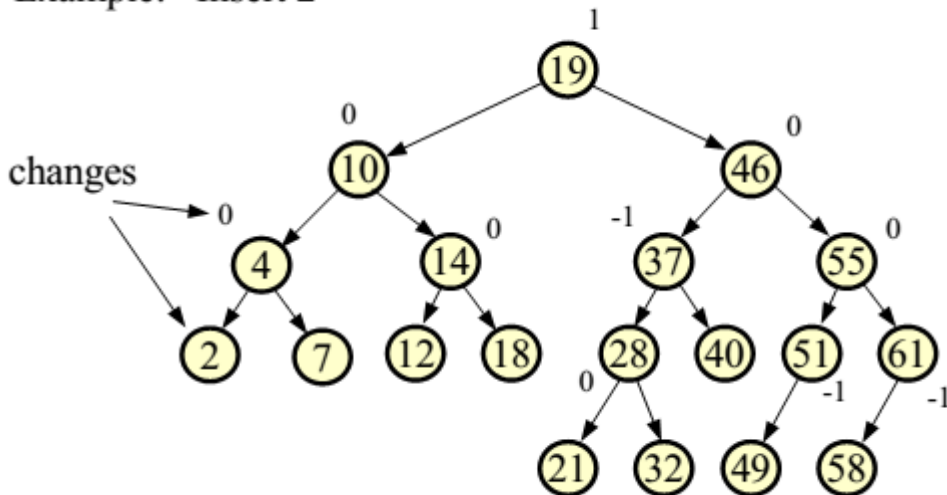


Plus mirror image of these three cases

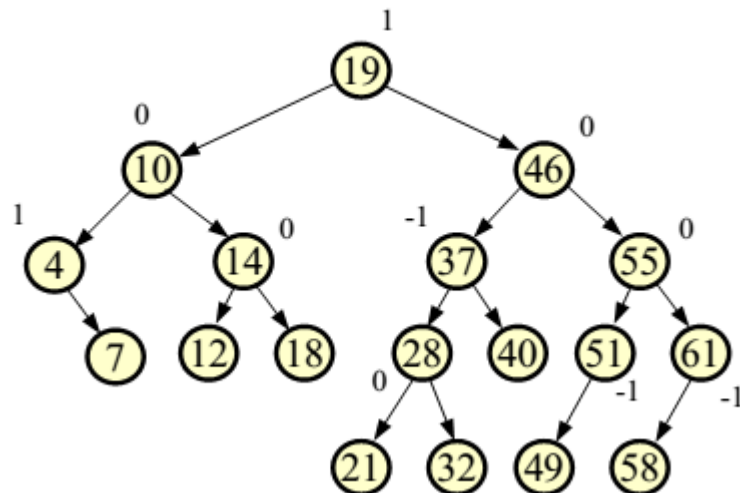
Example:



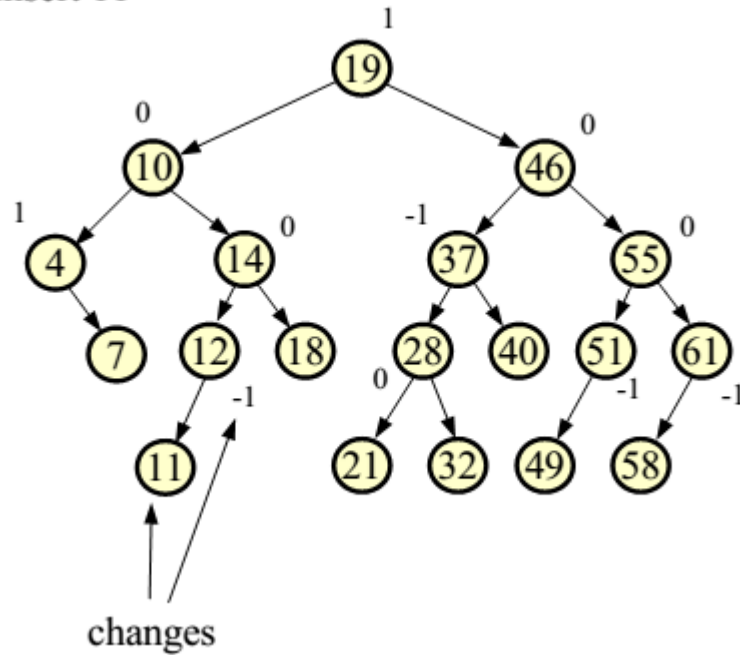
Example: Insert 2



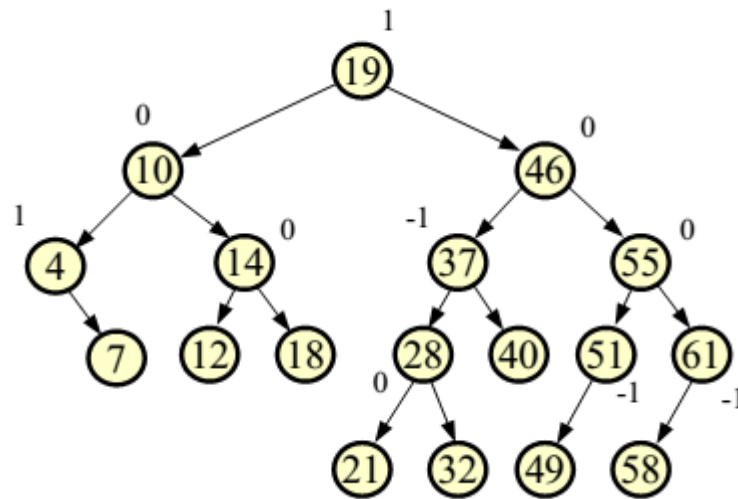
Example:



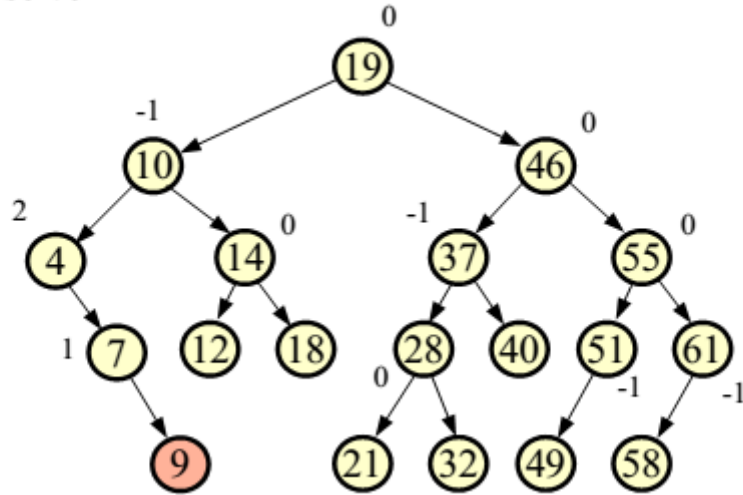
Example: Insert 11



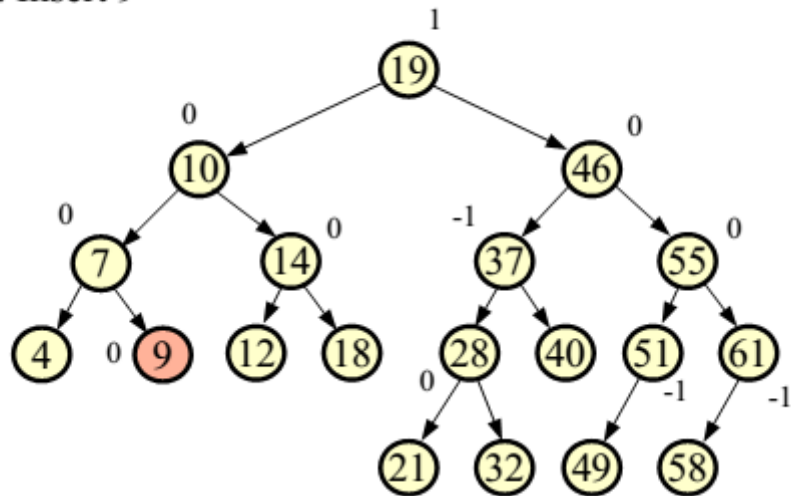
Example:



Example: Insert 9

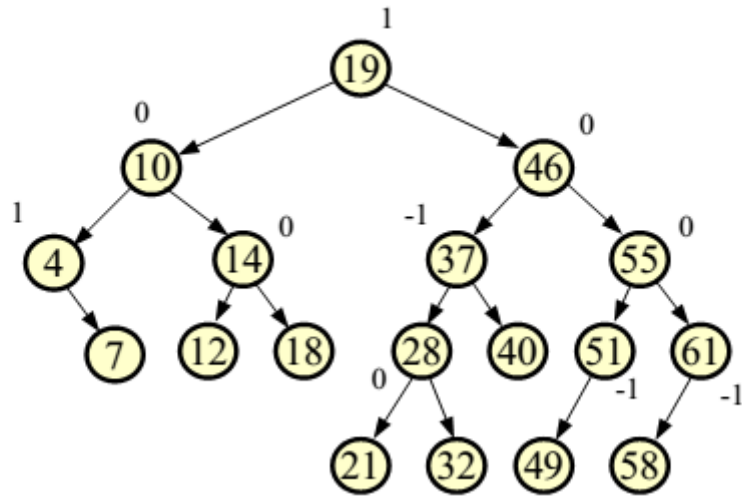


Example: Insert 9

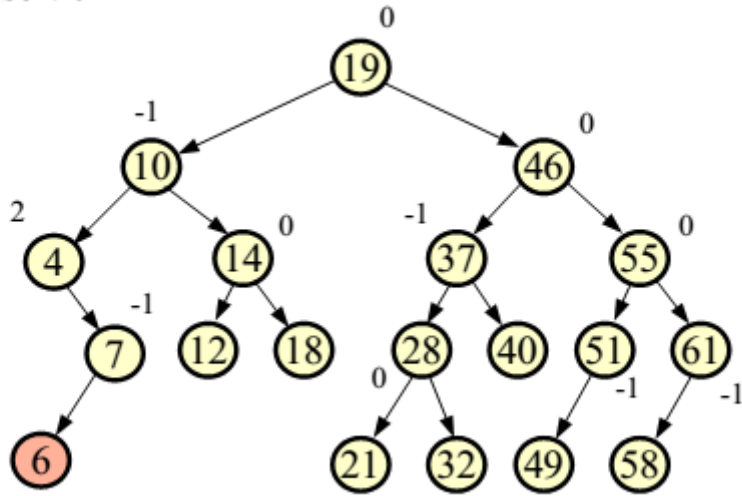


Rotation around 7

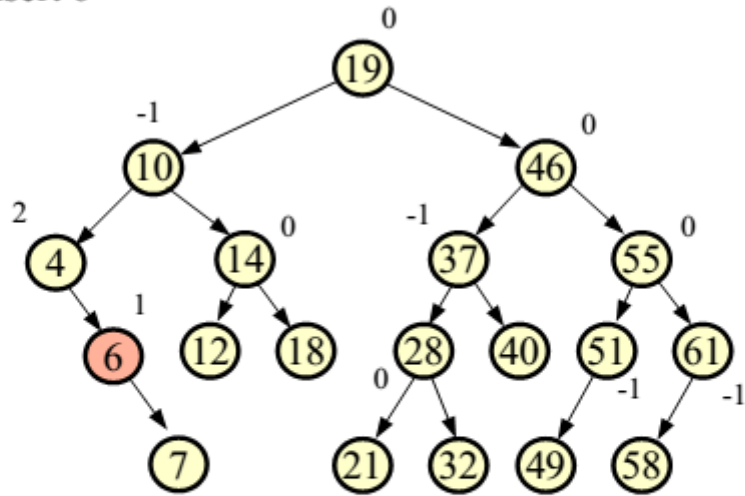
Example:



Example: Insert 6

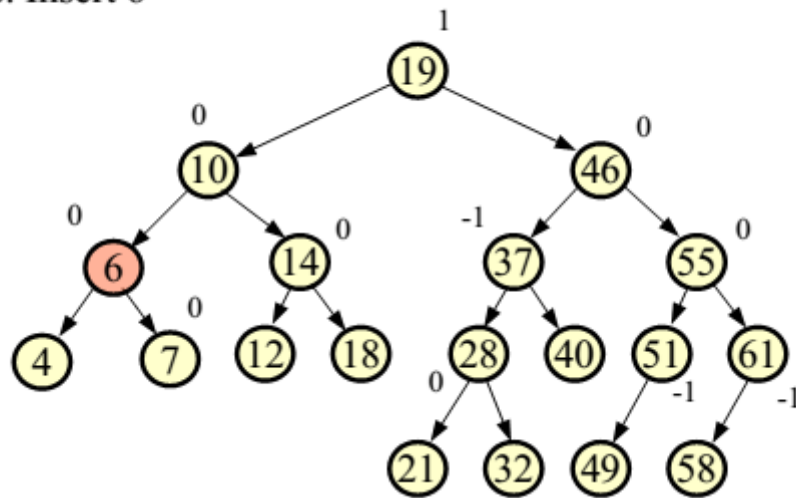


Example: Insert 6



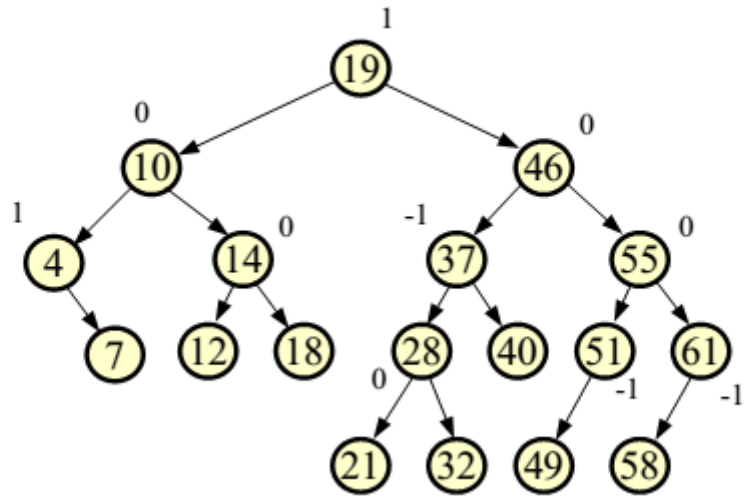
Double rotation

Example: Insert 6

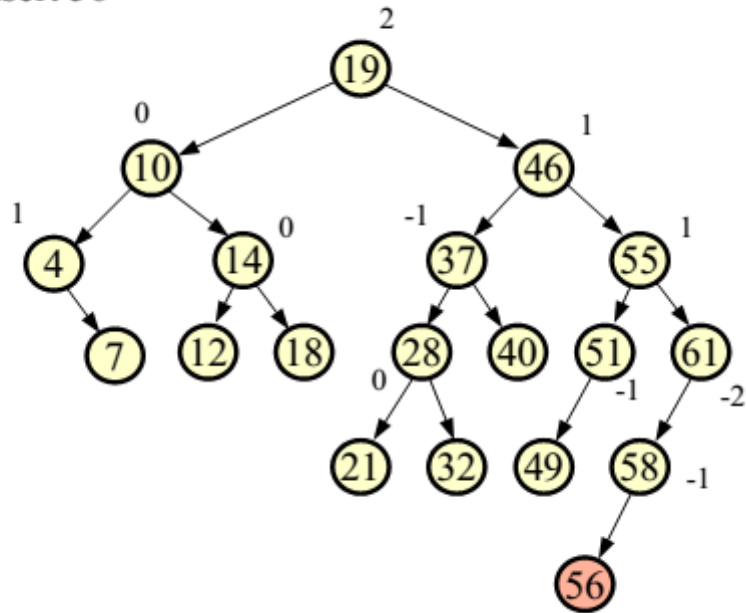


Double rotation

Example:

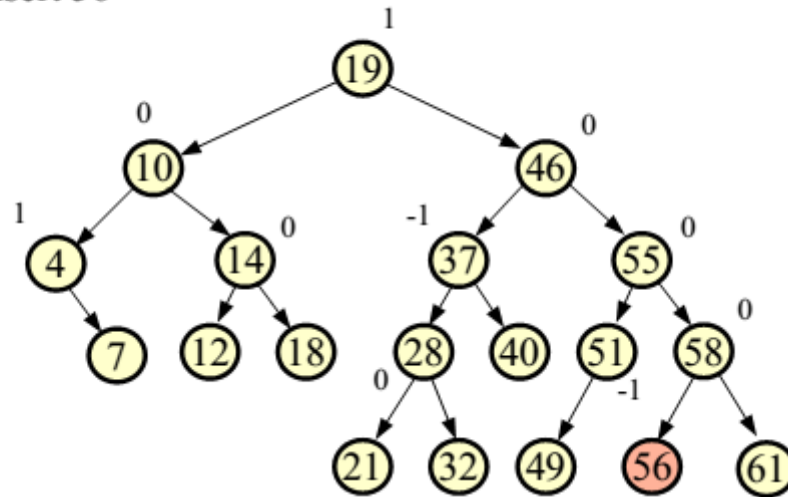


Example: Insert 56



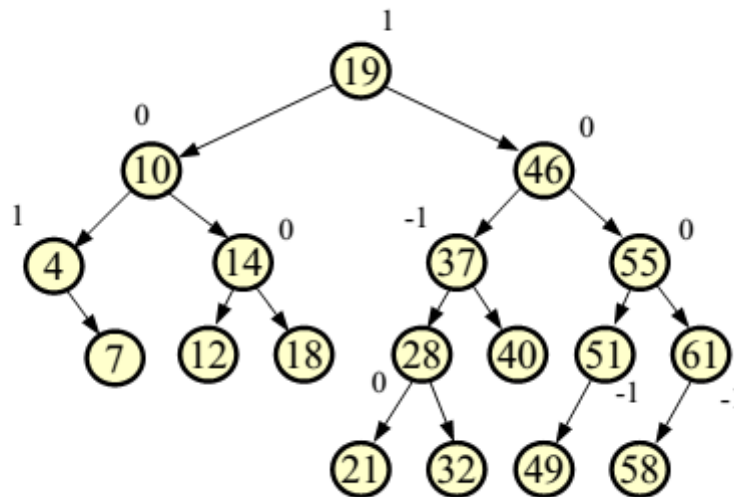


Example: Insert 56

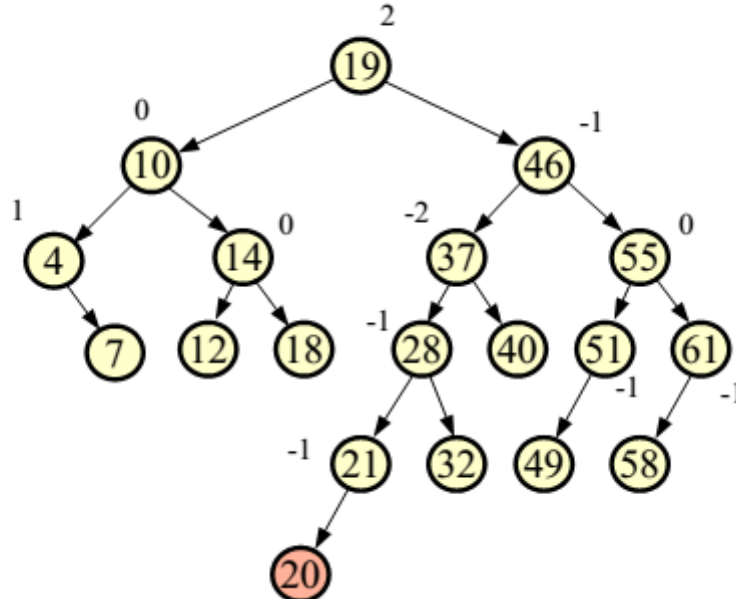


Single rotation around 58

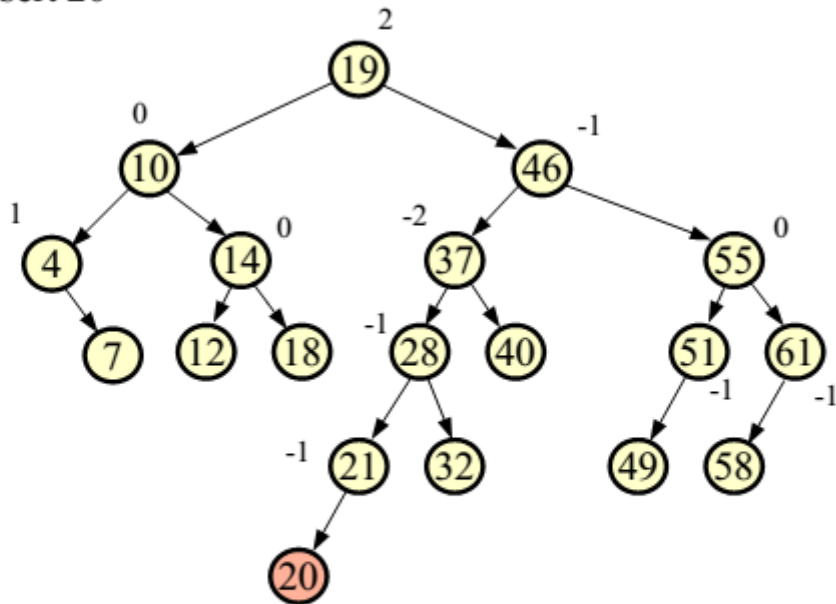
Example:



Example: Insert 20

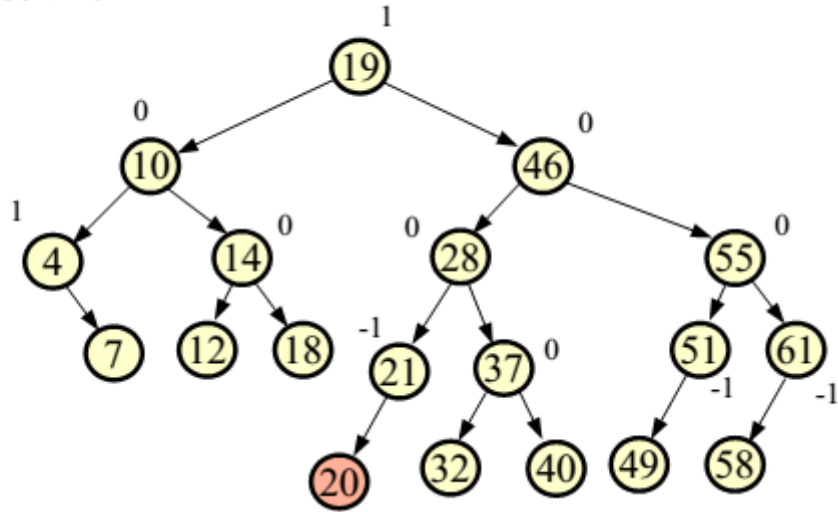


Example: Insert 20



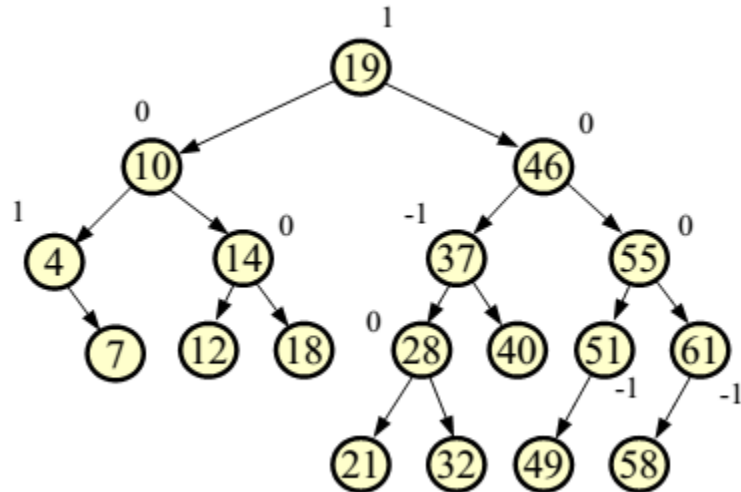
Rotate around 28

Example: Insert 20

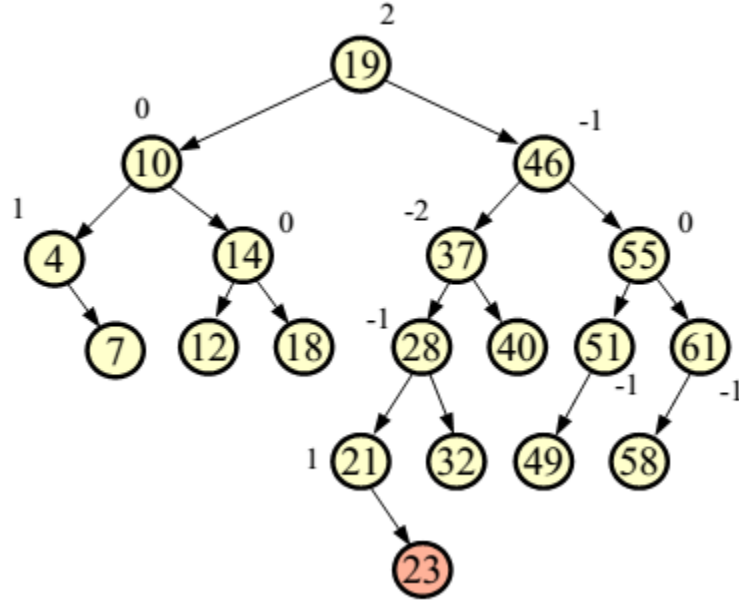


Rotate around 28

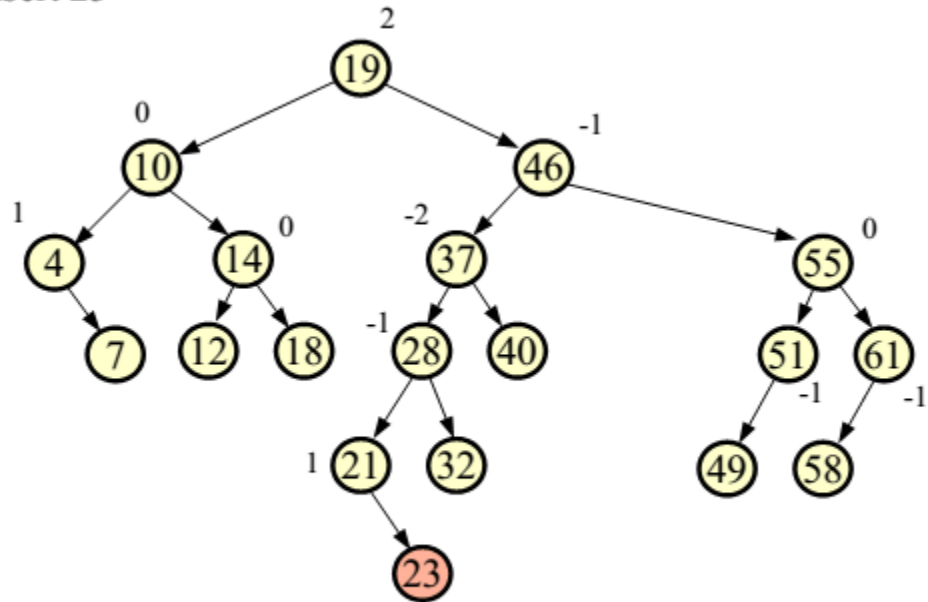
Example:



Example: Insert 23

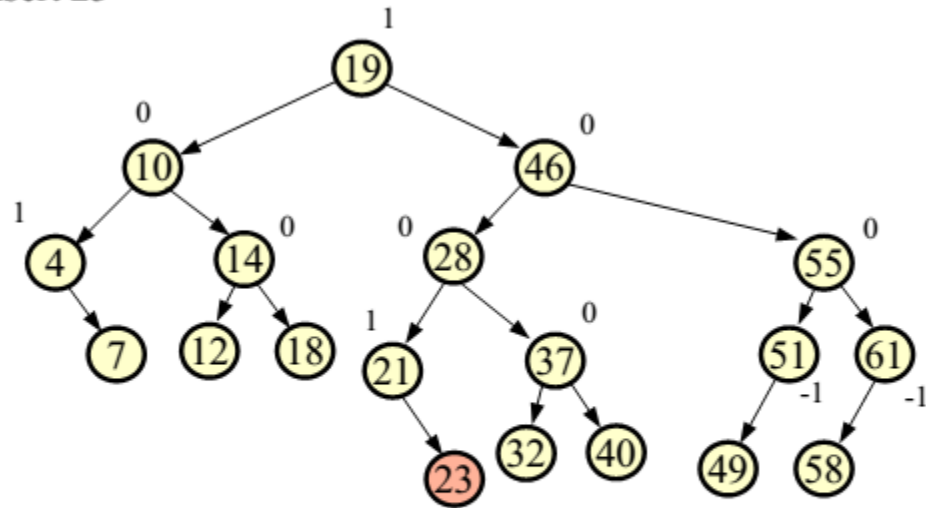


Example: Insert 23



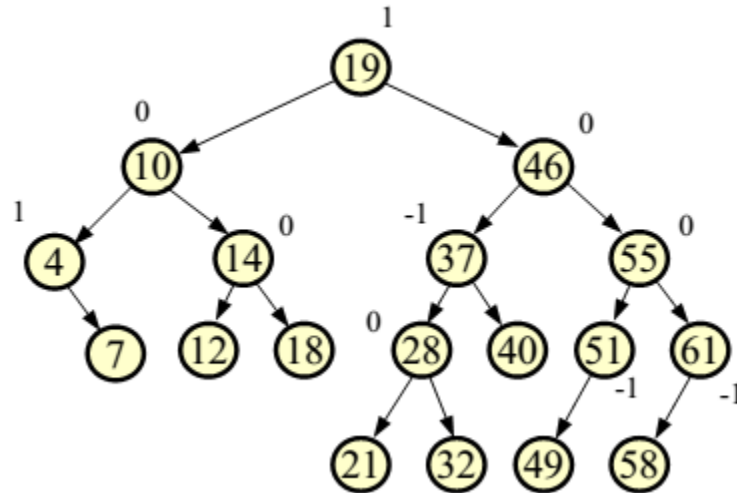
Rotation around 28

Example: Insert 23

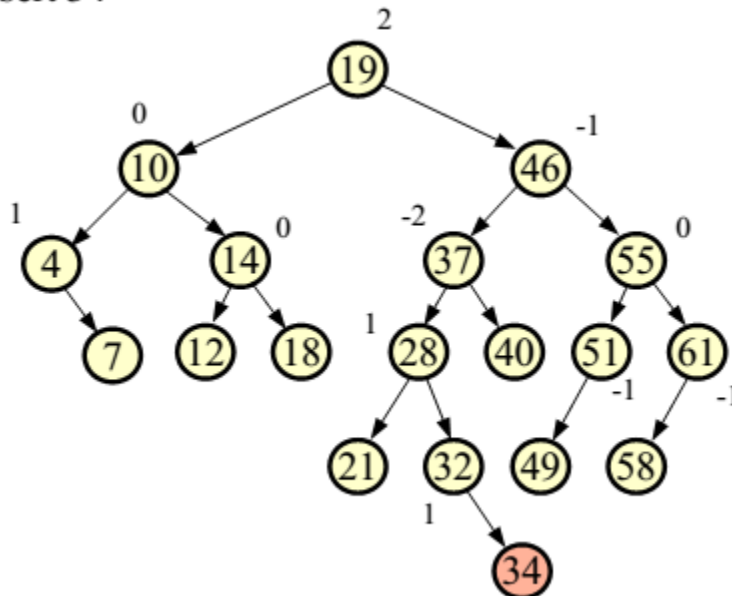


Rotation around 28

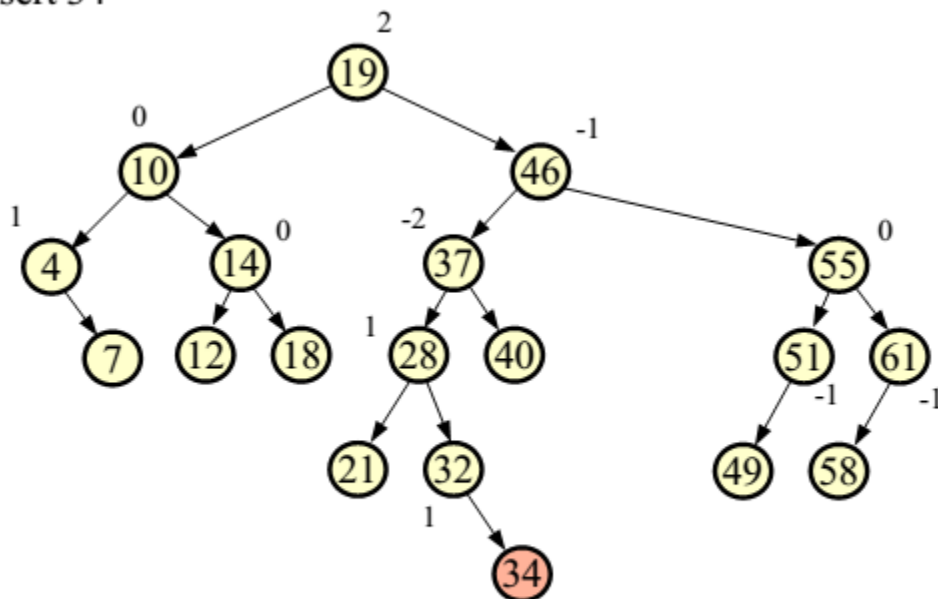
Example:



Example: Insert 34

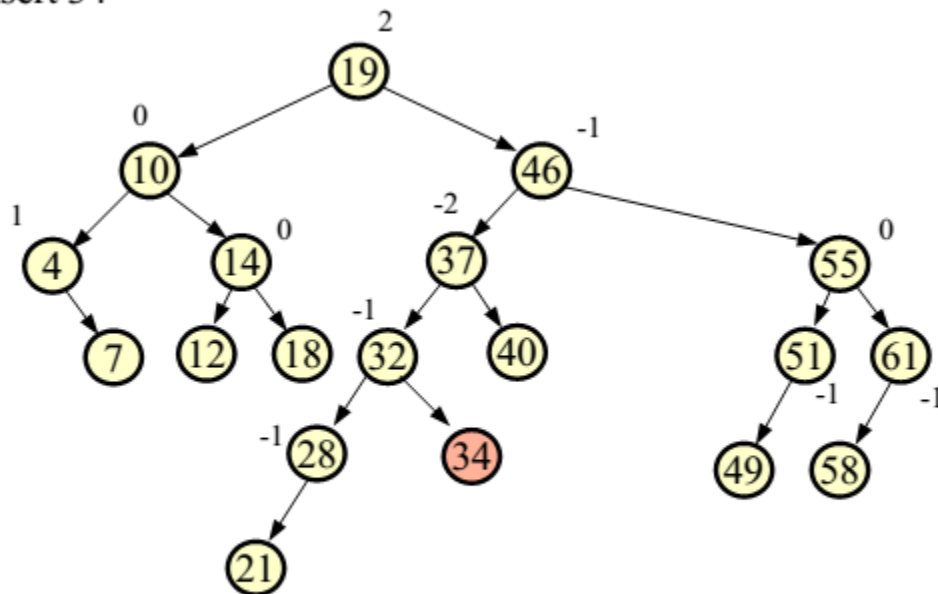


Example: Insert 34



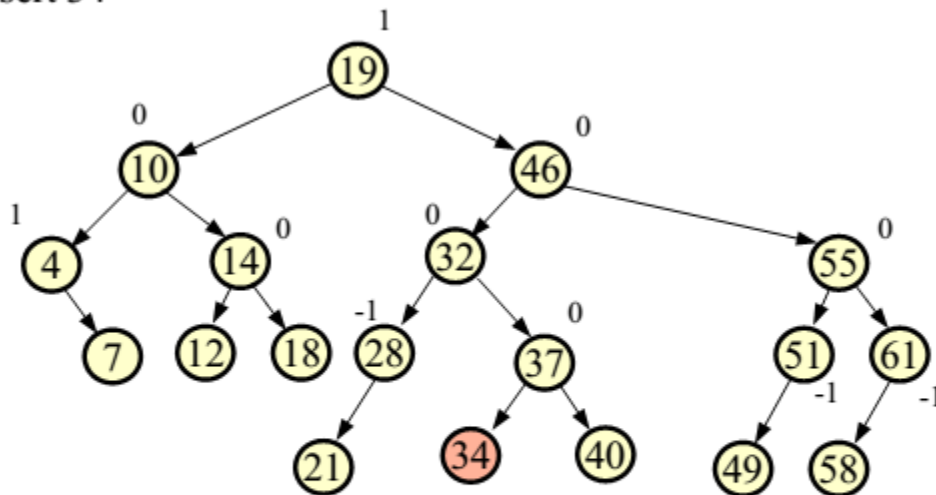
Double rotation around 32

Example: Insert 34



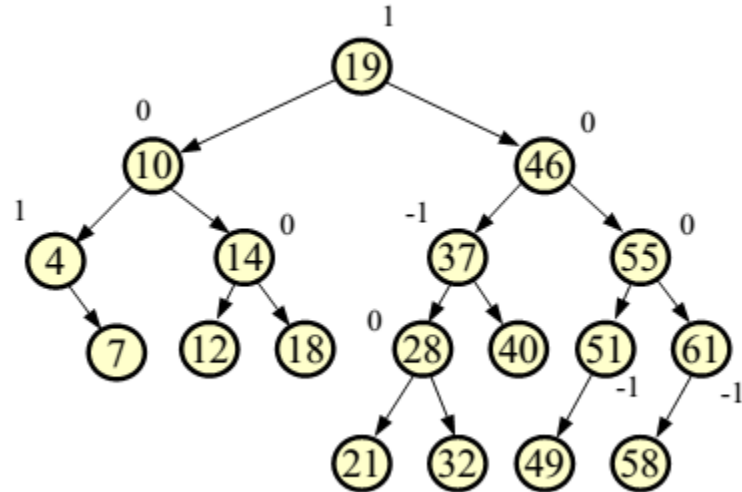
Double rotation around 32

Example: Insert 34

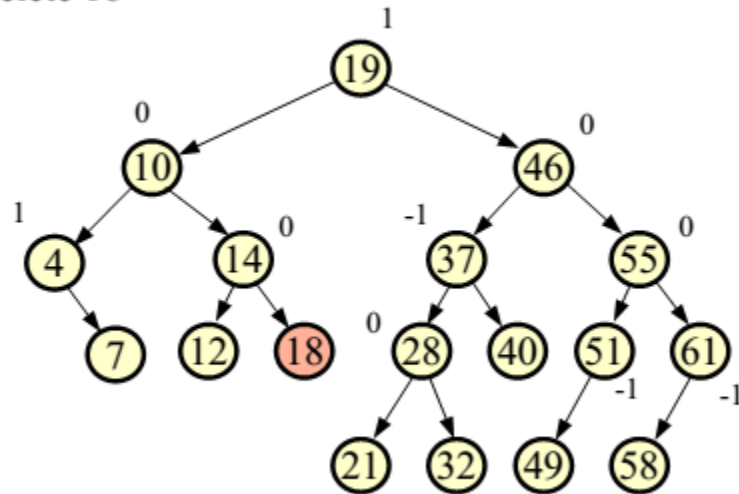


Double rotation around 32

Example:

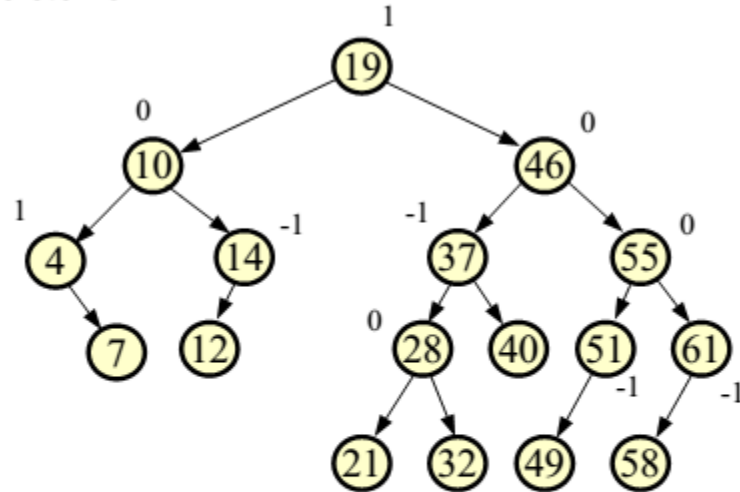


Example: Delete 18



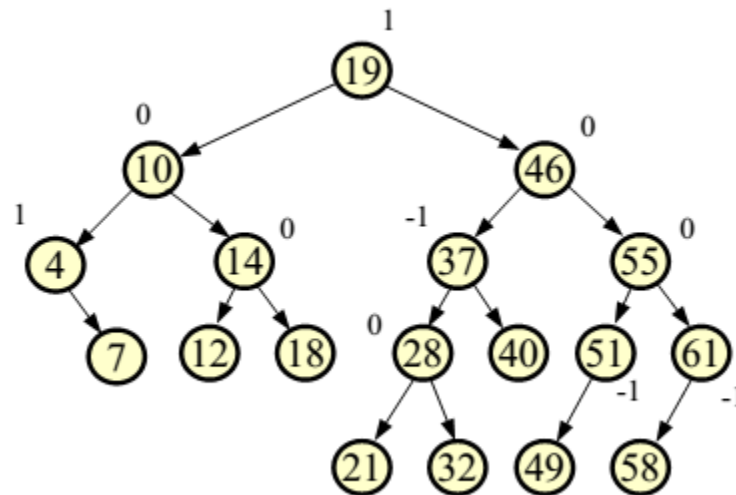


Example: Delete 18

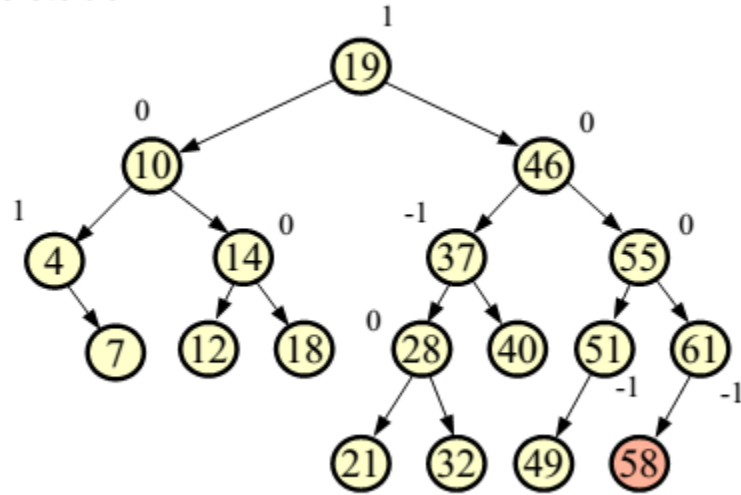


No change

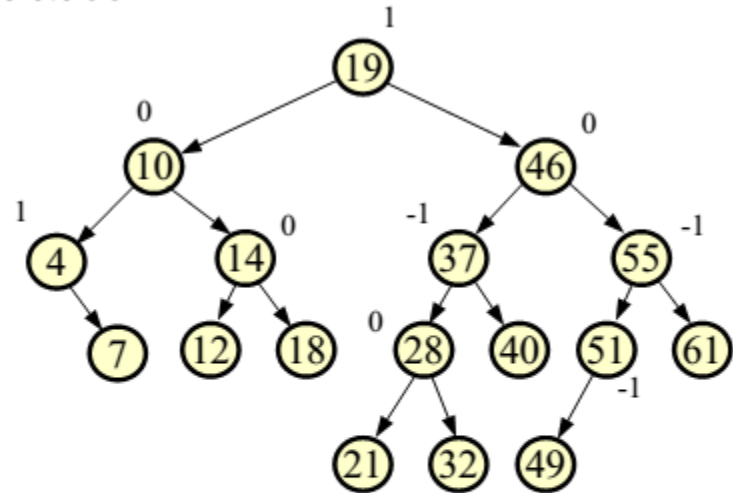
Example:



Example: Delete 58

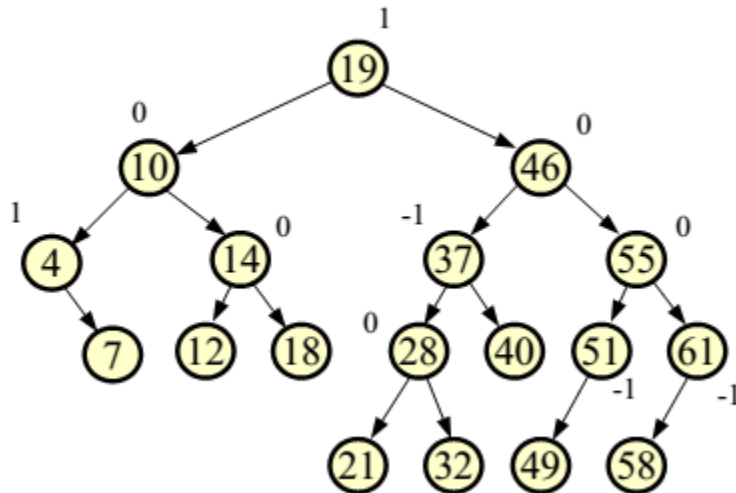


Example: Delete 58

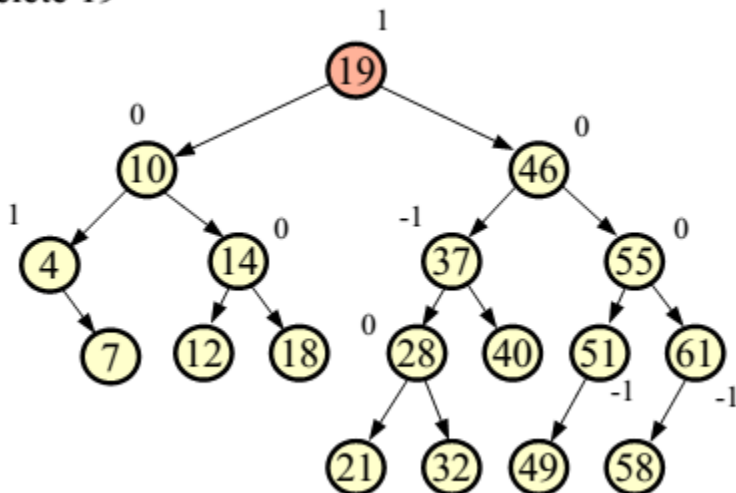


No change

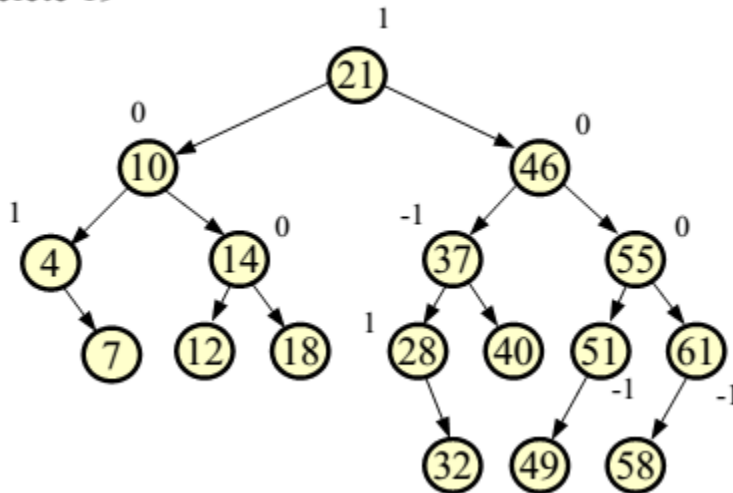
Example:



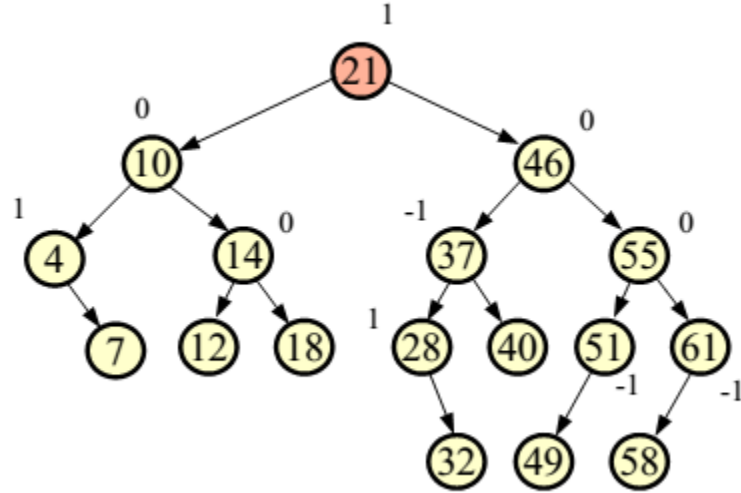
Example: Delete 19



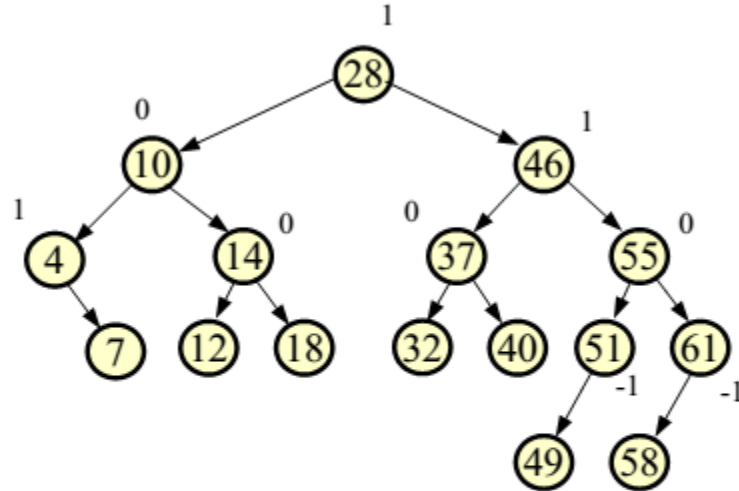
Example: Delete 19



Example: Delete 21



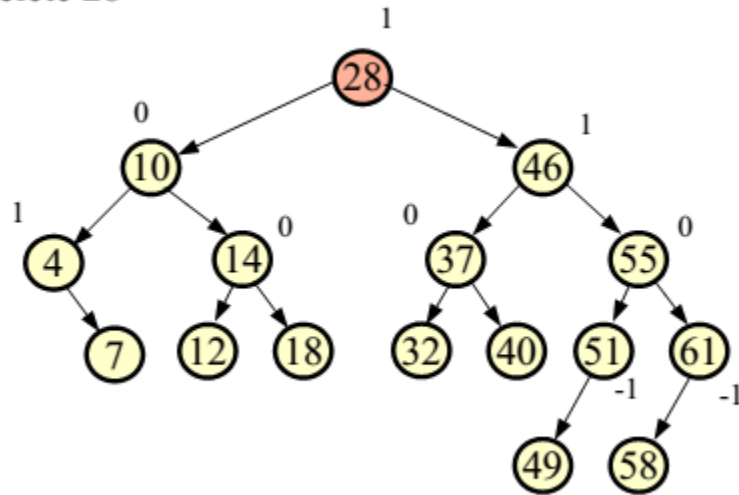
Example: Delete 21



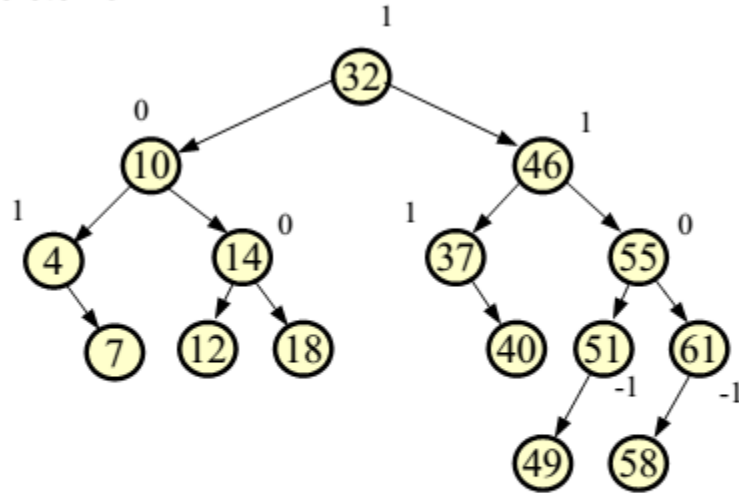
No change



Example: Delete 28

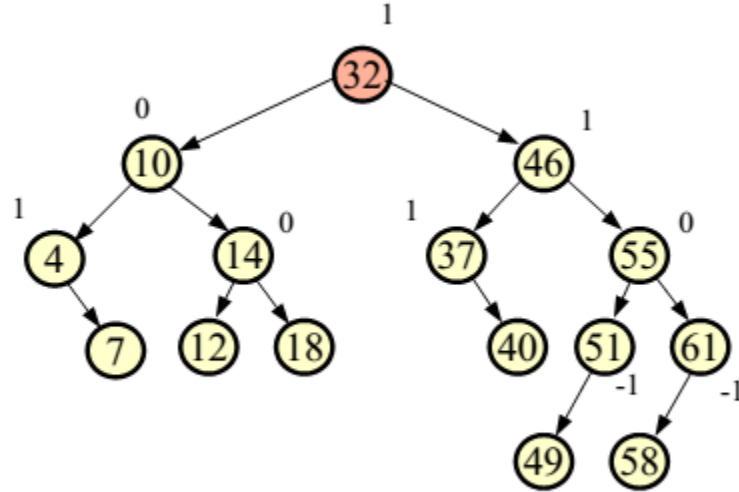


Example: Delete 28

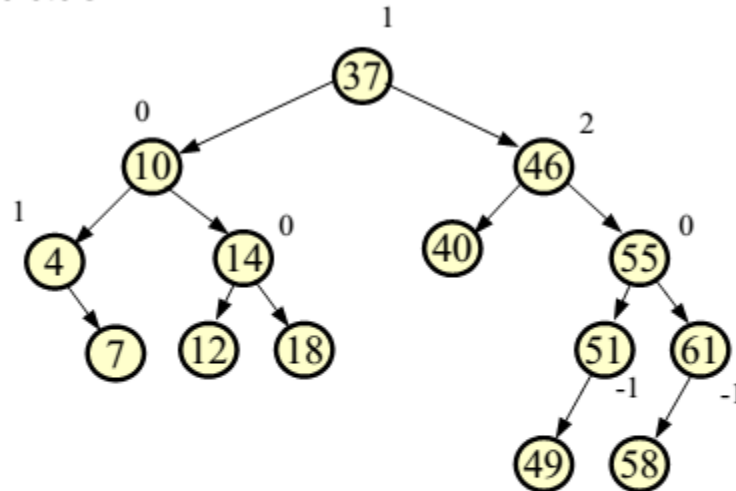


No change

Example: Delete 32

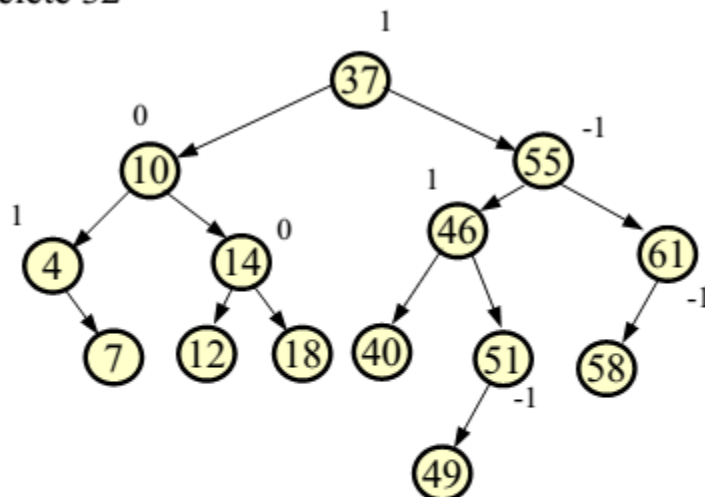


Example: Delete 32



Rotation around 55

Example: Delete 32



Rotation around 55